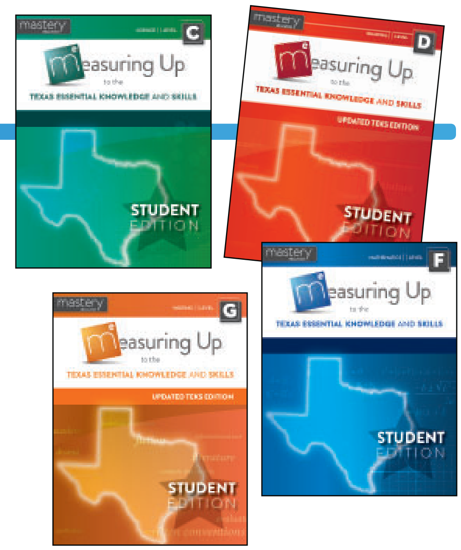
Measuring Up to the TEKS Sample Pack

Mathematics | Grade 8 | Lessons 6, 36, 37



The sample pack features:

- 3 full student lessons with complete Teacher Edition lessons
- 1 full Table of Contents for your grade level
- Lesson Correlations

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- ✔ Collaborative learning
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- ✔ Demonstration of problem-solving logic
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Lesson 6

Algebraic Representations of Dilations

R 8.3(C) Use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation.

Understand the TEKS

A **dilation** of a figure is an **enlargement** or a **reduction** that creates an **image** similar to the original figure. The ratio of an original side length to its corresponding image side length is constant. This ratio is the **scale factor**.

Dilations on a coordinate grid are performed by multiplying each *x*-coordinate and *y*-coordinate by the scale factor. This can be represented algebraically by $(x, y) \rightarrow (sx, sy)$, where s is the scale factor.

Words to Know
dilation
enlargement
reduction
image
scale factor

Guided Instruction

Problem 1

Transformation $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$ is applied to quadrilateral *FGHJ*, which has coordinates $F(-3, -1)$, $G(-6, 2)$, $H(1, 1)$, and $J(4, -3)$. What are the coordinates of $F'G'H'J'$?

Multiply the coordinates of the vertices by the scale factor.

Step 1 Determine the transformation.

A dilation is represented as $(x, y) \rightarrow (sx, sy)$, where s is the scale factor. Because this transformation is shown as $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$, it is a dilation with a scale factor $\frac{1}{2}$.

Step 2 Write the coordinates of the vertices of *FGHJ*.

F ____________, *G* ____________, *H* ____________, *J* ____________

Step 3 Multiply the coordinates of the vertices of *FGHJ* by the scale factor to find the coordinates of the vertices of $F'G'H'J'$.

$F(-3, -1)$ $F\left[\left(-3 \cdot \frac{1}{2}\right), \left(-1 \cdot \frac{1}{2}\right)\right]$ ________________

$G(-6, 2)$ $G\left[\left(-6 \cdot \frac{1}{2}\right), \left(2 \cdot \frac{1}{2}\right)\right]$ ________________

$H(1, 1)$ $H\left[\left(1 \cdot \frac{1}{2}\right), \left(1 \cdot \frac{1}{2}\right)\right]$ ________________

$J(4, -3)$ $J\left[\left(4 \cdot \frac{1}{2}\right), \left(-3 \cdot \frac{1}{2}\right)\right]$ ________________

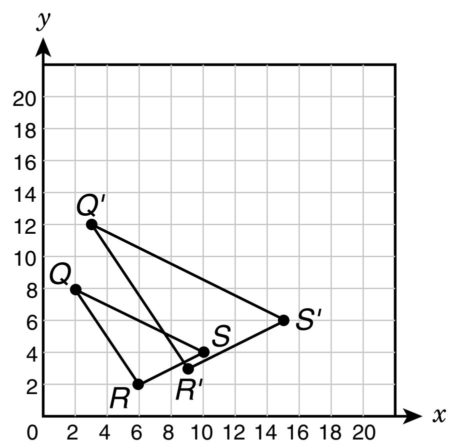
Solution

What are the coordinates of $F'G'H'J'$? ____

____

Problem 2

What is the algebraic rule that defines the dilation from $\triangle QRS$ to $\triangle Q'R'S'$? Present the answer in decimal form.



Determine the scale factor.

Step 1 Write the coordinates of the original figure.

Q ____________, R ____________, S ____________

Step 2 Write the coordinates of the image.

Q' ____________, R' ____________, S' ____________

Step 3 Determine the scale factor with a coordinate pair.

$Q(2, 8)$ $Q[(2 \cdot$ ____________$), (8 \cdot$ ____________$)]$ $Q'(3, 12)$

Step 4 Write the algebraic rule for the dilation.

$(x, y) \rightarrow ($____________$x,$ ____________$y)$

Solution

What is the algebraic rule that defines the dilation from $\triangle QRS$ to $\triangle Q'R'S'$?

____

Critical Thinking

Solve each problem.

Geography Connection

1. A geography class studies the locations of various countries in Africa using a poster-sized map. The students then take a test using a map shown on an 8.5- by 11-inch sheet of paper. Explain how the dilation $(x, y) \rightarrow (5x, 5y)$ could transform one map to the other if the same corner of each map were aligned as an origin on a coordinate grid. Which would be the original figure, and which would be the image?

Real World Connection

2. A local community is building a skate park on a rectangular plot of land. The skate area will be surrounded by seating. A contractor suggests that they eliminate some surrounding seating and change the original plans of the skate area by $(x, y) \rightarrow (\frac{1}{4}x, \frac{1}{4}y)$ so more people can skate at a time. Explain whether the contractor's suggestion is reasonable.

Collaborative Learning

3. Work with a partner to draw triangles on a coordinate grid with vertices at $(n, n + 1)$, $(n, n - 1)$, and $(n + 1, n - 1)$ for three different values of n. How does the value of n affect the original figures? How does it affect the final image when a dilation of $(x, y) \rightarrow (2x, 2y)$ is applied?

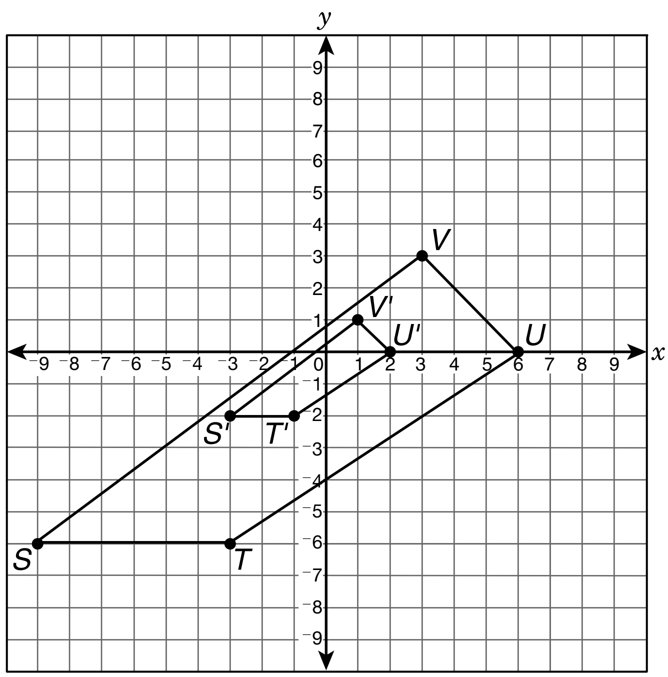
Elevate

4. Could you dilate a square on a coordinate grid as $(x, y) \rightarrow (0.5x, 0.5y)$ by instead applying $(x, y) \rightarrow (x - a, y - a)$ where $a = 0.5x$? Explain why or why not.

★ Practice

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 What values of m, n, p, *and* q would result in the dilation shown, where $(x, y) \rightarrow \left(\left(\frac{m}{n}\right)x, \left(\frac{p}{q}\right)y\right)$?



A $m = 2, n = 2, p = 6, q = 6$

B $m = 3, n = 9, p = 1, q = 3$

C $m = 12, n = 4, p = 12, q = 4$

D $m = 4, n = 16, p = 4, q = 16$

---

2 Two of the vertices of $\triangle SLM$ are at (5, 10) and (250, 500). The corresponding vertices of $\triangle KNR$ are at (0.2, 0.4) and (10, 20), but it is unknown which sets of coordinate pairs correspond with each other. Which of the following could be a rule of the dilation relating the triangles?

F $(x, y) \rightarrow (1{,}250x, 1{,}250y)$

G $(x, y) \rightarrow (0.5x, 0.5y)$

H $(x, y) \rightarrow (25x, 25y)$

J $(x, y) \rightarrow (2x, 2y)$

3 A dilation of $(x, y) \rightarrow (0.2x, 0.2y)$ is applied to a triangle. The image coordinates are $(4t, 3t)$, $(-7t, t)$, and $(10t, -2t)$. Which of the following is a coordinate of the original figure?

A $(50t, -10t)$

B $(0.8t, 0.6t)$

C $(-7.2t, 1.2t)$

D $(3.8t, 2.8t)$

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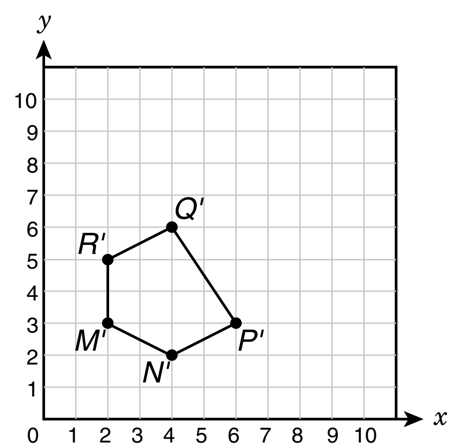
4 Point Q in figure $MNPQR$ is at $\left(\frac{1}{2}, \frac{3}{4}\right)$. The coordinate grid shows the image after dilation.



Which rule represents the dilation?

F $(x, y) \rightarrow (8x, 8y)$

G $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$

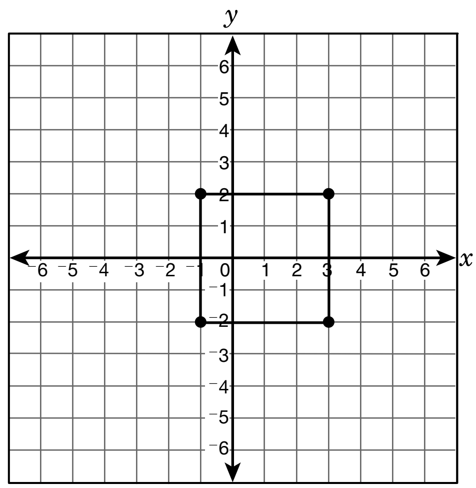
H $(x, y) \rightarrow \left(\frac{1}{8}x, \frac{1}{8}y\right)$

J $(x, y) \rightarrow \left(\frac{2}{3}x, \frac{2}{3}y\right)$

★ Assessment

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 The transformation rule $(x, y) \rightarrow (4x, 4y)$ is applied to the figure shown.



Which are the final coordinates?

A (0.75, 0.5), (0.75, −0.5), (−0.25, −0.5), (−0.25, 0.5)

B (12, 8), (12, −8), (−12, −8), (−12, 8)

C (12, 8), (12, −8), (−4, −8), (−4, 8)

D (0.75, 0.5), (0.75, −0.5), (−0.75, −0.5), (−0.75, 0.5)

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2 A figure has vertices (x, y), $(x + 2, y)$ and $(x + 6, y + 2)$. If it is dilated by $(x, y) \rightarrow (3x, 3y)$, what are the new coordinates?

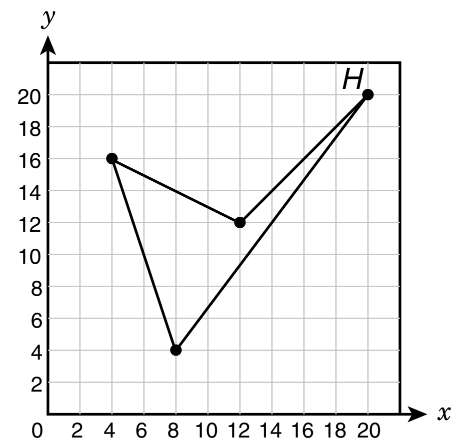
F $(3x, 3y)$, $(3x + 6, 3y)$, $(3x + 18, 3y + 6)$

G $(x + 3, y + 3)$, $(x + 5, y + 3)$, $(x + 9, y + 5)$

H $(3x, 3y)$, $(3x + 2, 3y)$, $(3x + 6, 3y + 2)$

J $(3x, 3y)$, $(3x, 3y)$, $(3x, 3y)$, $(3x, 3y)$

3 If $(x, y) \rightarrow (4ax, 4ay)$, for what value of a would the rightmost point of the resulting image have an x-value of 10?



A $\frac{4}{10}$

B $\frac{1}{8}$

C $\frac{1}{2}$

D $\frac{10}{4}$

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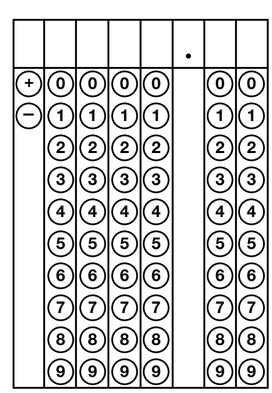
4 Three dilations are applied to a figure in the order shown.
$(x, y) \rightarrow (0.8x, 0.8y)$
$(x, y) \rightarrow (0.06x, 0.06y)$
$(x, y) \rightarrow (20x, 20y)$

What value of r provides a single rule that would give the same image as the three rules shown? Write your answer in decimal form. $(x, y) \rightarrow (rx, ry)$

Record your answer and fill in the bubbles on the following grid. Be sure to use the correct place value.



Lesson 36 Properties of Transformations

Ⓢ 8.10(A) Generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

Ⓢ 8.10(B) Differentiate between transformations that preserve congruence and those that do not.

Understand the TEKS

Words to Know
transformations
congruence
orientation

The **transformations** that describe movement of a figure are translations, reflections, rotations, and dilations.

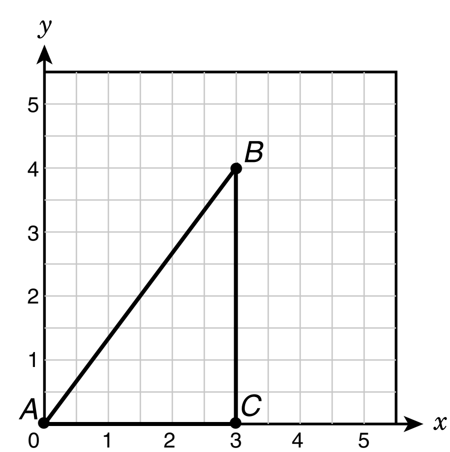
Translations, reflections, and rotations are **congruence** transformations because the pre-image and image are congruent. While a reflection results in congruence, it is the only transformation that does not preserve **orientation**, or the order of the vertices.

Dilations with a scale factor other than 1 result in an enlargement or a reduction of the figure, so this is not a congruence transformation. The difference in the effect on congruence can be recognized in the algebraic form of the transformations. Dilations are the only transformations that incorporate a scale factor other than 1 or -1 in the x-coordinate and y-coordinate of the image.

Guided Instruction

Problem 1

Triangle ABC undergoes a transformation on the coordinate grid. What are the potential effects on congruence and orientation?



Step 1 Analyze the effects of a translation.

A translation would move the triangle according to $(x, y) \rightarrow (x + a, y + b)$.

The resulting image ____________ be congruent and the orientation will be ____________.

Step 2 Analyze the effects of a reflection.

A reflection would move the triangle according to $(x, y) \rightarrow (-x, y)$ or $(x, y) \rightarrow (x, -y)$.

The resulting image ____________ be congruent and the orientation will be ____________.

Step 3 Analyze the effects of a rotation.

A rotation would move the triangle according to $(x, y) \rightarrow (y, -x)$, $(x, y) \rightarrow (-x, -y)$), $(x, y) \rightarrow (-y, x)$), or $(x, y) \rightarrow (x, y)$.

The resulting image ____________ be congruent and the orientation will be ____________.

Step 4 Analyze the effects of a dilation.

A dilation would move the triangle according to $(x, y) \rightarrow (sx, sy)$.

The resulting image ____________ be congruent, if the dilation factor has a value other than 1, and the orientation will be ____________.

Solution

What are the potential effects on congruence and orientation?

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Problem 2

A transformation is performed on a figure such that $(x, y) \rightarrow (ax, by)$. What must be true for the transformation to preserve congruence?

Step 1 Each transformation that preserves congruence has an algebraic rule with coefficients of ±____________ for x and y. So, $|a|$ must have a value of ____________.

Step 2 Notice that you do not need to state a relationship between a and b, because a dilation is the only transformation that does not preserve congruence, and if b was a positive number other than 1, a would be also.

Solution

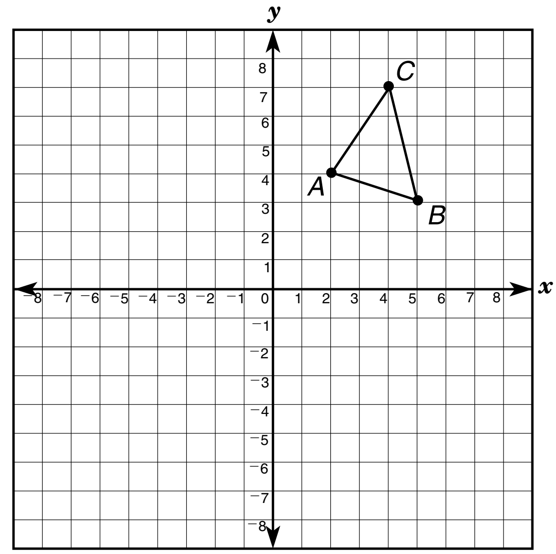
What must be true for the transformation to preserve congruence?

____

Critical Thinking

Solve each problem.

Elevate **1.** Consider $\triangle ABC$ with vertices at $A(2, 4)$, $B(5, 3)$, $C(4, 7)$. First, graph $\triangle A'B'C'$, the reflection of $\triangle ABC$ across the y-axis. Then, graph $\triangle A''B''C''$, the reflection of $\triangle ABC$ across the x-axis. Describe how the image triangles are like the original triangle, and how they are different.



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Real World Connection **2.** An architect is designing the layout of a house on a coordinate grid that represents a plot of land. She has the floor plan drawn with the living room on the northeast side of the house. She decides which part of the house is facing north without changing the orientation of the house. What transformation can she do? Explain your reasoning.

____

____

____

____

Discuss **3.** Work with a partner to analyze whether a translation followed by a dilation with a scale factor not equal to 1 could have the same final image as a rotation. Use algebraic representations of the transformations to explain your reasoning. Record examples and findings in your math journal. Share your results with the class.

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____

★ Practice

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 A figure is plotted in Quadrant I on a coordinate grid. Which of the following could result in a figure that is in the same location and looks identical to the original figure? Note: $a \neq 0$ and $b \neq 0$.

A $(x, y) \rightarrow (x + a, y + b)$

B $(x, y) \rightarrow (-x, y)$

C $(x, y) \rightarrow (ax, ay)$

D $(x, y) \rightarrow (-x, -y)$

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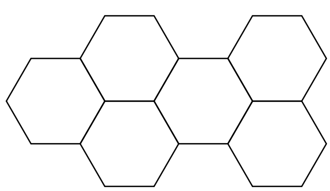
2 A regular tessellation is a collection of one or more congruent regular polygons with no gaps or overlaps.



Which transformation could not be used to create the tessellation?

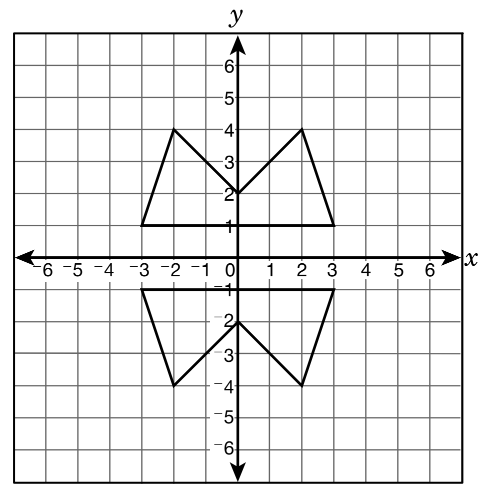
F Dilation

G Reflection

H Translation

J Rotation

3 The coordinate grid shows the pre-image and image of a transformation.



Which of the following statements must be true about the figures?

A The figures may or may not be congruent.

B The orientation must be different.

C The orientation must be the same.

D The figures must be congruent.

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4 What effect does a reflection have on the perimeter and area of a triangle?

F The perimeter must change.

G The area must stay the same.

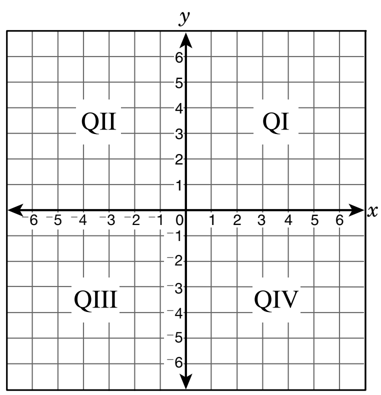
H The perimeter may increase.

J The area may decrease.

★ Assessment

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 Which algebraic representation could be a transformation that takes the pre-image from Quadrant IV to an image that is congruent, has the same orientation, and is in the same quadrant? Note: $a > 1$ and $b \neq 0$.



A $(x, y) \rightarrow (-y, x)$

B $(x, y) \rightarrow (-x, y)$

C $(x, y) \rightarrow (ax, ay)$

D $(x, y) \rightarrow (x + a, y + b)$

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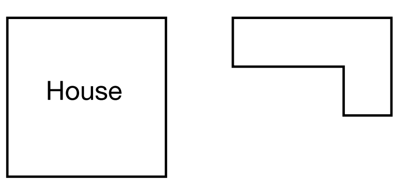
2 Jerome is buying fencing to build a dog run in his backyard as shown.



He decides to move the dog run according to the transformation $(x, y) \rightarrow (y, -x)$. How is the cost of his project material affected?

F The cost increases.

G The cost decreases.

H The cost stays the same.

J The values for x and y must be known to determine the effect.

3 A figure has vertices labeled alphabetically from M, in the counterclockwise direction. Which single transformations result in an image with vertices labeled alphabetically from M in the clockwise direction?

I. Rotation 180° clockwise
II. Rotation 270° clockwise
III. Reflection across the x-axis
IV. Reflection across the y-axis

A I only

B III and IV only

C I and II only

D I, II, III, and IV

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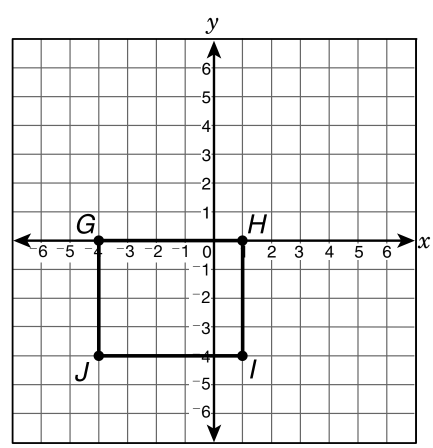
4 Rhonda claims that she can reflect the rectangle across the y-axis to get an identical rectangle to the one created by the transformation $(x, y) \rightarrow (x + 3, y + 0)$.



Is she correct? Why or why not?

F Yes, the rectangles are congruent with identical orientation.

G No, one rectangle changes orientation.

H No, the rectangles are not congruent.

J No, the rectangles are not congruent and have different orientations.

Lesson 37 Scatterplots

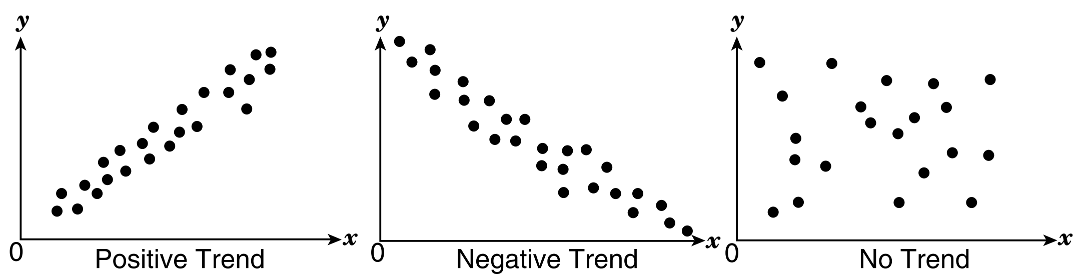
8.11(A) Construct a scatterplot and describe the observed data to address questions of association such as linear, non-linear, and no association between bivariate data.

8.11(C) Simulate generating random samples of the same size from a population with known characteristics to develop the notion of a random sample being representative of the population from which it was selected.

Understand the TEKS

Words to Know
scatterplot
trend
association
correlation
linear association
random sample
population

Scatterplots help us determine whether there is a relationship between two quantities. A scatterplot is a graph of individual data points relating two different types of values (such as width and height) for a set of data. A pattern of data points on a scatterplot is called a **trend**, an **association**, or a **correlation**. Sometimes the data points of a scatterplot may be approximated by a straight line, in which case the quantities have a **linear association**. If the slope of this line is positive, the association between the two variables is positive. If the slope is negative, the association is negative. If the data points in a scatterplot cluster closely to the line, the association is considered strong. If the data points vary in distance from the general line, the association is considered weak.
If the data can be approximated by a line that is not straight, the association is nonlinear. Some data sets can show no trend between the two variables.



Statistical data is often represented using scatterplots. When very large populations are studied, such as all teenagers in the United States, **random samples** are taken. Random sampling enables a **population** to be studied that would otherwise be unreasonable to study due to the large sample size. It is not reasonable to survey all teenagers in the United States about their favorite musical artist. Random sampling within the population is much more feasible as a members of the population are chosen randomly to provide data for a study. Random sampling provides accurate results for an entire population given that the sample taken is truly random and sufficiently large in number.

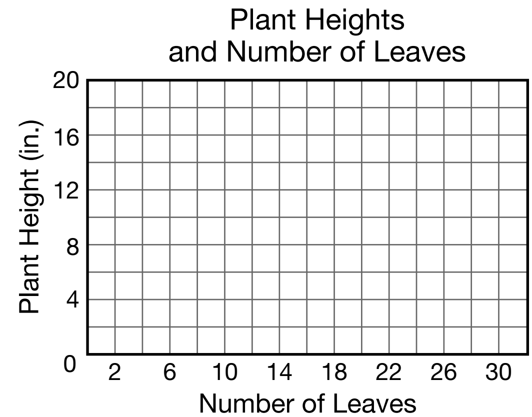
Guided Instruction

Problem

Mr. Connor's science class is studying plant growth. They want to want to learn more about the relationship between the height of a plant and the number of leaves on the plant. They measured the height of 11 plants, counted the number of leaves on each plant, and recorded the data in the table below. Plot the data in the table. What is the relationship between the height of the plant and the number of leaves?

Step 1 Plot the data points.

Number of Leaves	Height of Plant (in.)
10	4
11	7
13	9
14	5
15	8
16	7
16	11
19	9
19	14
20	8
25	12



Step 2 Interpret the scatterplot. The data points show that plant height tends to increase as the number of leaves ____________.

Step 3 Determine if the association is linear or nonlinear.

Do the points form a reasonably straight line? ____________

Step 4 Determine if the association is weak or strong.

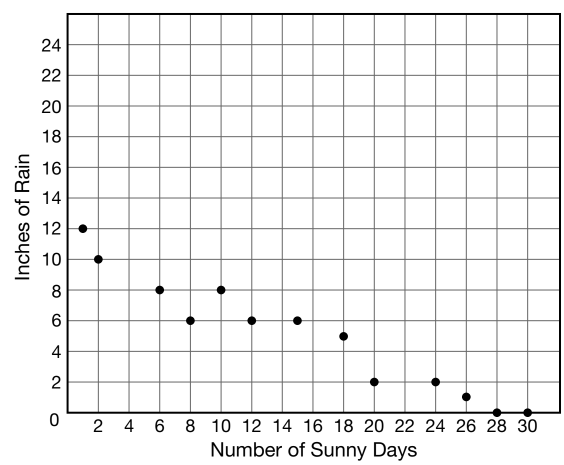
The dots are clustered closely to form a line, so the association is____________.

Solution

What is the relationship between the height of the plant and the number of leaves?
The data show a ____________, ____________, and ____________ trend.

Another Example

The scatterplot shows the number of sunny days and the number of inches of rainfall in a city during one month in different years.



Determine whether there is an association between the variables. If so, describe the association.

About how many inches of rain would you expect to fall in a year that has 4 sunny days in the month?

As the number of days of sunshine increases, the number of inches of rainfall tends to ____________.

The dots form a ____________ line, so the association is ____________.

Look at the number of inches of rainfall for fewer than 4 sunny days and more than 4 sunny days.

When there were 2 sunny days, there were ____________ inches of rainfall.

When there were 6 sunny days, there were ____________ inches of rainfall.

If there were 4 days of sunshine, there would be between ____________ inches and ____________ inches of rain.

Critical Thinking

Solve each problem.

Collaborative Learning

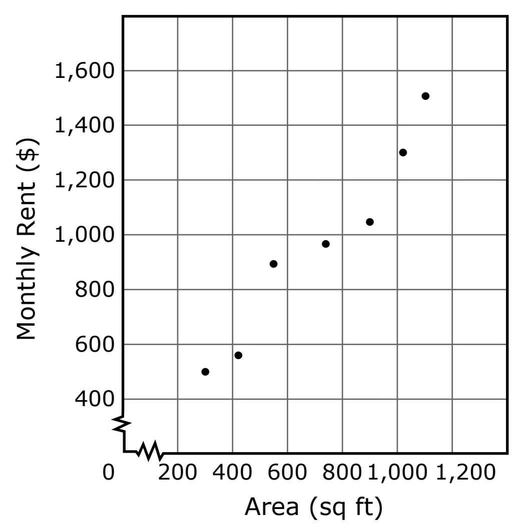
1. Work in groups of 2 or 3 as directed by your teacher. Survey your classmates. Ask each classmate for their height and shoe size. Create a scatterplot to show the data. Determine if an association exists between a student's height and shoe size. If so, describe the association. Can you use your scatterplot to make any conclusions about a population? Explain.

Elevate

2. The scatterplot at the right shows the relationship between the sizes (in square feet) of apartments in a certain neighborhood of a city and the price of those apartments.

Describe any association shown in the scatterplot.

Using the scatterplot, predict the cost of an apartment that has an area of 500 square feet.
Show or explain your work.



Media Connection

3. Search the Internet to find an example of a published scatterplot. On a poster, describe what data the scatterplot represents, what trends it shows, and three predictions that you can make from the scatterplot. Use colored markings to show how you found your predictions based on the graph. Write one quiz question about your scatterplot, with the answer on the other side, so that other students may quiz themselves using your graph.

Elevate

4. The association shown in a scatterplot can be described as strong or weak. What would a strong association look like? What would a weak association look like? Explain.

Lesson 36

Properties of Transformations

8.10(A) Generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

8.10(B) Differentiate between transformations that preserve congruence and those that do not.

Understand the TEKS

The **transformations** that describe movement of a figure are translations, reflections, rotations, and dilations.

Translations, reflections, and rotations are **congruence** transformations because the pre-image and image are congruent. While a reflection results in congruence, it is the only transformation that does not preserve **orientation**, or the order of the vertices.

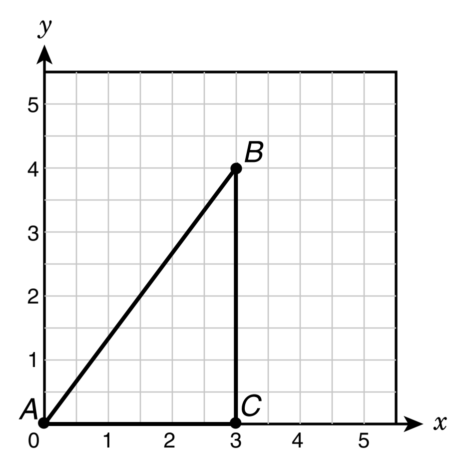
Words to Know
transformations
congruence
orientation

Dilations with a scale factor other than 1 result in an enlargement or a reduction of the figure, so this is not a congruence transformation. The difference in the effect on congruence can be recognized in the algebraic form of the transformations. Dilations are the only transformations that incorporate a scale factor other than 1 or -1 in the x-coordinate and y-coordinate of the image.

Guided Instruction

Problem 1

Triangle ABC undergoes a transformation on the coordinate grid. What are the potential effects on congruence and orientation?



Step 1 Analyze the effects of a translation.

A translation would move the triangle according to $(x, y) \rightarrow (x + a, y + b)$.

The resulting image ____________ be congruent and the orientation will be ____________.

Step 2 Analyze the effects of a reflection.

A reflection would move the triangle according to $(x, y) \rightarrow (-x, y)$ or $(x, y) \rightarrow (x, -y)$.

The resulting image ____________ be congruent and the orientation will be ____________.

Step 3 Analyze the effects of a rotation.

A rotation would move the triangle according to $(x, y) \rightarrow (y, -x)$, $(x, y) \rightarrow (-x, -y)$), $(x, y) \rightarrow (-y, x)$), or $(x, y) \rightarrow (x, y)$.

The resulting image ____________ be congruent and the orientation will be ____________.

Step 4 Analyze the effects of a dilation.

A dilation would move the triangle according to $(x, y) \rightarrow (sx, sy)$.

The resulting image ___________ be congruent, if the dilation factor has a value other than 1, and the orientation will be ___________.

Solution

What are the potential effects on congruence and orientation?

____

____

____

____

Problem 2

A transformation is performed on a figure such that $(x, y) \rightarrow (ax, by)$. What must be true for the transformation to preserve congruence?

Step 1 Each transformation that preserves congruence has an algebraic rule with coefficients of ±___________ for x and y. So, $|a|$ must have a value of ___________.

Step 2 Notice that you do not need to state a relationship between a and b, because a dilation is the only transformation that does not preserve congruence, and if b was a positive number other than 1, a would be also.

Solution

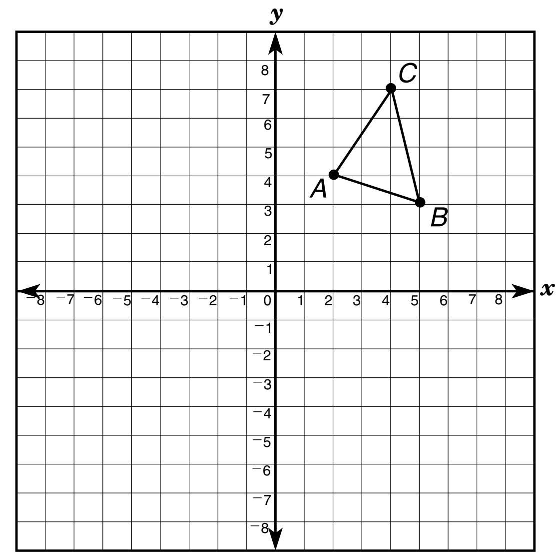
What must be true for the transformation to preserve congruence?

____

Critical Thinking

Solve each problem.

Elevate **1.** Consider $\triangle ABC$ with vertices at $A(2, 4)$, $B(5, 3)$, $C(4, 7)$. First, graph $\triangle A'B'C'$, the reflection of $\triangle ABC$ across the y-axis. Then, graph $\triangle A''B''C''$, the reflection of $\triangle ABC$ across the x-axis. Describe how the image triangles are like the original triangle, and how they are different.



Real World Connection **2.** An architect is designing the layout of a house on a coordinate grid that represents a plot of land. She has the floor plan drawn with the living room on the northeast side of the house. She decides which part of the house is facing north without changing the orientation of the house. What transformation can she do? Explain your reasoning.

Discuss **3.** Work with a partner to analyze whether a translation followed by a dilation with a scale factor not equal to 1 could have the same final image as a rotation. Use algebraic representations of the transformations to explain your reasoning. Record examples and findings in your math journal. Share your results with the class.

★ Practice

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 A figure is plotted in Quadrant I on a coordinate grid. Which of the following could result in a figure that is in the same location and looks identical to the original figure? Note: $a \neq 0$ and $b \neq 0$.

A $(x, y) \rightarrow (x + a, y + b)$

B $(x, y) \rightarrow (-x, y)$

C $(x, y) \rightarrow (ax, ay)$

D $(x, y) \rightarrow (-x, -y)$

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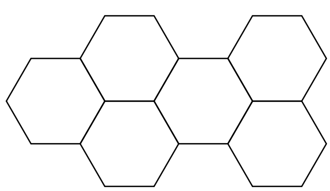
2 A regular tessellation is a collection of one or more congruent regular polygons with no gaps or overlaps.



Which transformation could not be used to create the tessellation?

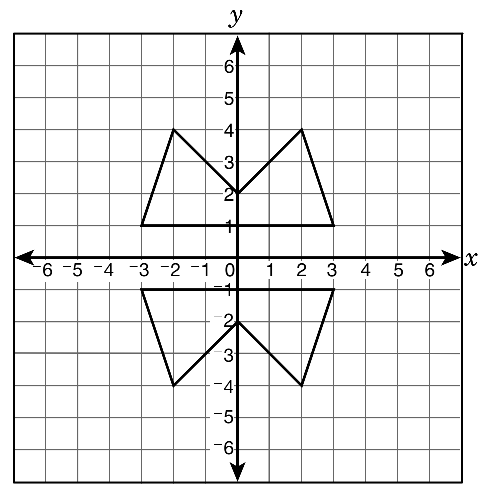
F Dilation

G Reflection

H Translation

J Rotation

3 The coordinate grid shows the pre-image and image of a transformation.



Which of the following statements must be true about the figures?

A The figures may or may not be congruent.

B The orientation must be different.

C The orientation must be the same.

D The figures must be congruent.

---

4 What effect does a reflection have on the perimeter and area of a triangle?

F The perimeter must change.

G The area must stay the same.

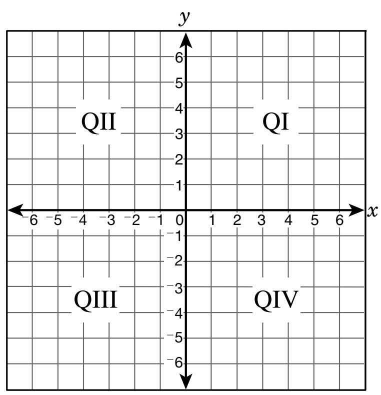
H The perimeter may increase.

J The area may decrease.

★ Assessment

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 Which algebraic representation could be a transformation that takes the pre-image from Quadrant IV to an image that is congruent, has the same orientation, and is in the same quadrant? Note: $a > 1$ and $b \neq 0$.



A $(x, y) \rightarrow (-y, x)$

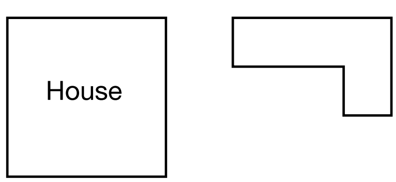
B $(x, y) \rightarrow (-x, y)$

C $(x, y) \rightarrow (ax, ay)$

D $(x, y) \rightarrow (x + a, y + b)$

---

2 Jerome is buying fencing to build a dog run in his backyard as shown.



He decides to move the dog run according to the transformation $(x, y) \rightarrow (y, -x)$. How is the cost of his project material affected?

F The cost increases.

G The cost decreases.

H The cost stays the same.

J The values for x and y must be known to determine the effect.

3 A figure has vertices labeled alphabetically from *M*, in the counterclockwise direction. Which single transformations result in an image with vertices labeled alphabetically from *M* in the clockwise direction?

I. Rotation 180° clockwise
II. Rotation 270° clockwise
III. Reflection across the x-axis
IV. Reflection across the y-axis

A I only

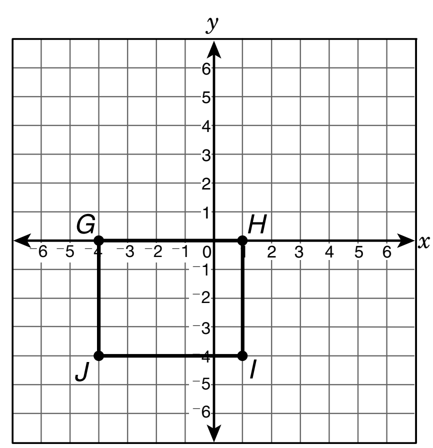
B III and IV only

C I and II only

D I, II, III, and IV

---

4 Rhonda claims that she can reflect the rectangle across the y-axis to get an identical rectangle to the one created by the transformation $(x, y) \rightarrow (x + 3, y + 0)$.



Is she correct? Why or why not?

F Yes, the rectangles are congruent with identical orientation.

G No, one rectangle changes orientation.

H No, the rectangles are not congruent.

J No, the rectangles are not congruent and have different orientations.

Lesson 37 Scatterplots

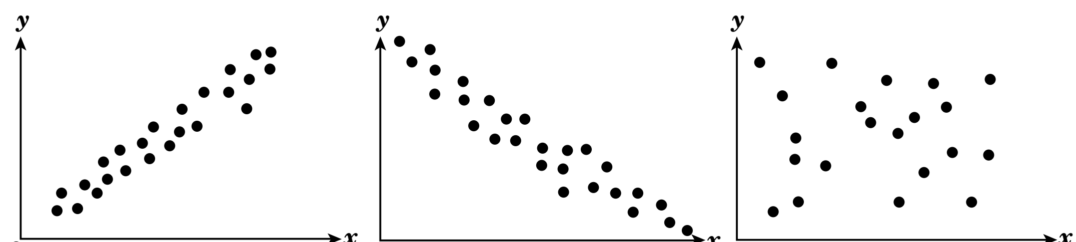
S 8.11(A) Construct a scatterplot and describe the observed data to address questions of association such as linear, non-linear, and no association between bivariate data.

8.11(C) Simulate generating random samples of the same size from a population with known characteristics to develop the notion of a random sample being representative of the population from which it was selected.

Understand the TEKS

Words to Know
- scatterplot
- trend
- association
- correlation
- linear association
- random sample
- population

Scatterplots help us determine whether there is a relationship between two quantities. A scatterplot is a graph of individual data points relating two different types of values (such as width and height) for a set of data. A pattern of data points on a scatterplot is called a **trend**, an **association**, or a **correlation**. Sometimes the data points of a scatterplot may be approximated by a straight line, in which case the quantities have a **linear association**. If the slope of this line is positive, the association between the two variables is positive. If the slope is negative, the association is negative. If the data points in a scatterplot cluster closely to the line, the association is considered strong. If the data points vary in distance from the general line, the association is considered weak.
If the data can be approximated by a line that is not straight, the association is nonlinear. Some data sets can show no trend between the two variables.



Positive Trend   Negative Trend   No Trend

Statistical data is often represented using scatterplots. When very large populations are studied, such as all teenagers in the United States, **random samples** are taken. Random sampling enables a **population** to be studied that would otherwise be unreasonable to study due to the large sample size. It is not reasonable to survey all teenagers in the United States about their favorite musical artist. Random sampling within the population is much more feasible as a members of the population are chosen randomly to provide data for a study. Random sampling provides accurate results for an entire population given that the sample taken is truly random and sufficiently large in number.

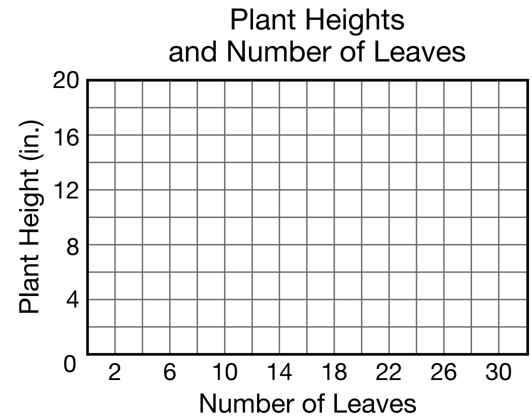
Guided Instruction

Problem

Mr. Connor's science class is studying plant growth. They want to want to learn more about the relationship between the height of a plant and the number of leaves on the plant. They measured the height of 11 plants, counted the number of leaves on each plant, and recorded the data in the table below. Plot the data in the table. What is the relationship between the height of the plant and the number of leaves?

Step 1 Plot the data points.

Number of Leaves	Height of Plant (in.)
10	4
11	7
13	9
14	5
15	8
16	7
16	11
19	9
19	14
20	8
25	12



Step 2 Interpret the scatterplot. The data points show that plant height tends to increase as the number of leaves ____________.

Step 3 Determine if the association is linear or nonlinear.

Do the points form a reasonably straight line? ____________

Step 4 Determine if the association is weak or strong.

The dots are clustered closely to form a line, so the association is____________.

Solution

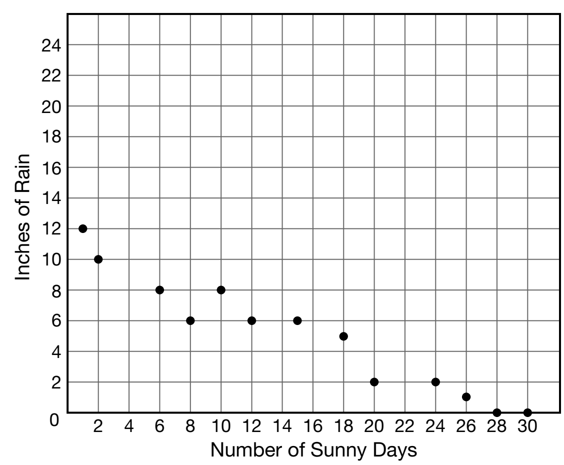
What is the relationship between the height of the plant and the number of leaves?
The data show a ____________, ____________, and ____________ trend.

Another Example

The scatterplot shows the number of sunny days and the number of inches of rainfall in a city during one month in different years.

Determine whether there is an association between the variables. If so, describe the association.

About how many inches of rain would you expect to fall in a year that has 4 sunny days in the month?



As the number of days of sunshine increases, the number of inches of rainfall tends to ____________.

The dots form a ____________ line, so the association is ____________.

Look at the number of inches of rainfall for fewer than 4 sunny days and more than 4 sunny days.

When there were 2 sunny days, there were ____________ inches of rainfall.

When there were 6 sunny days, there were ____________ inches of rainfall.

If there were 4 days of sunshine, there would be between ____________ inches and ____________ inches of rain.

Critical Thinking

Solve each problem.

Collaborative Learning

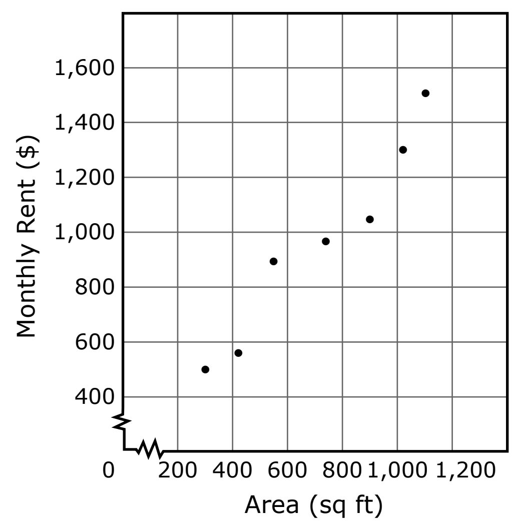
1. Work in groups of 2 or 3 as directed by your teacher. Survey your classmates. Ask each classmate for their height and shoe size. Create a scatterplot to show the data. Determine if an association exists between a student's height and shoe size. If so, describe the association. Can you use your scatterplot to make any conclusions about a population? Explain.

Elevate

2. The scatterplot at the right shows the relationship between the sizes (in square feet) of apartments in a certain neighborhood of a city and the price of those apartments.

Describe any association shown in the scatterplot.

Using the scatterplot, predict the cost of an apartment that has an area of 500 square feet.
Show or explain your work.



Media Connection

3. Search the Internet to find an example of a published scatterplot. On a poster, describe what data the scatterplot represents, what trends it shows, and three predictions that you can make from the scatterplot. Use colored markings to show how you found your predictions based on the graph. Write one quiz question about your scatterplot, with the answer on the other side, so that other students may quiz themselves using your graph.

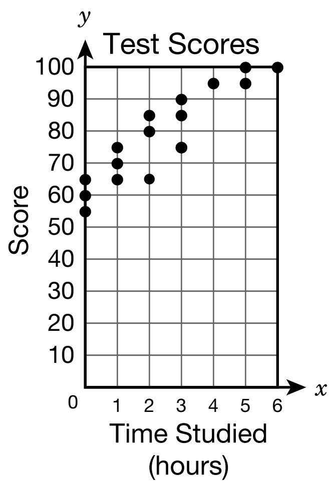
Elevate

4. The association shown in a scatterplot can be described as strong or weak. What would a strong association look like? What would a weak association look like? Explain.

★ Practice

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 The scatterplot below shows the test scores and number of hours spent studying for 16 students.



Which statement best describes the correlation between the study times and the test scores?

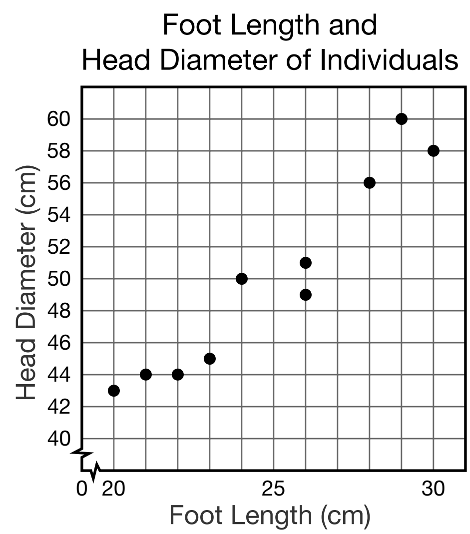
A There is a strong positive correlation between the time spent studying and the test score.

B There is a negative correlation between the time spent studying and the test score.

C There is a weak positive correlation between the time spent studying and the test score.

D There is a weak negative correlation between the time spent studying and the test score.

2 The scatterplot below shows the relationship between foot length and head diameter of individuals.



If a person has a head diameter of approximately 54 centimeters, what would be the most reasonable foot length for that individual based on the data provided?

F 21 cm

G 23 cm

H 27 cm

J 30 cm

---

3 If you graphed data that compared the price of a movie ticket to annual sales for that movie, what would you expect the scatterplot to show?

A A strong positive correlation

B A weak negative correlation

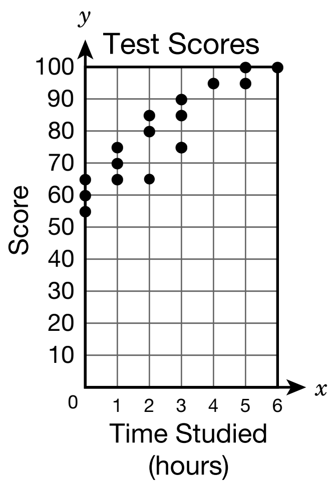
C A weak positive correlation

D No correlation

★ Practice

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 The scatterplot below shows the test scores and number of hours spent studying for 16 students.



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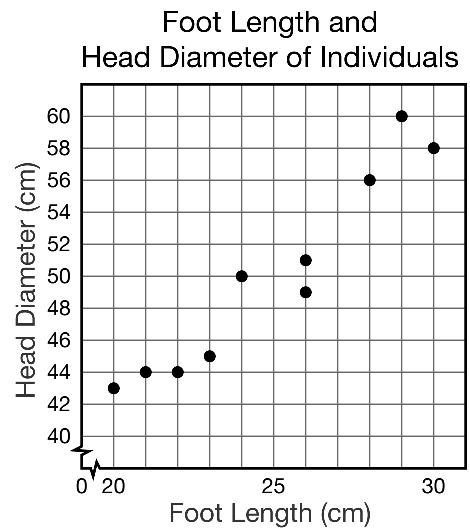

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F 21 cm

G 23 cm

H 27 cm

J 30 cm

---

3 If you graphed data that compared the price of a movie ticket to annual sales for that movie, what would you expect the scatterplot to show?

A A strong positive correlation

B A weak negative correlation

C A weak positive correlation

D No correlation

★ Assessment

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 A scatterplot shows the relationship between the value of a rare comic book, in dollars, and the number of years since it was purchased. What would you expect the scatterplot to look like?

A The data points are clustered tightly around a straight line with a positive slope.

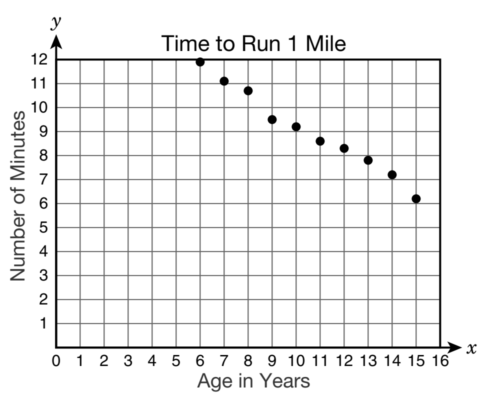
B The data points are clustered tightly around a straight line with a negative slope.

C The data points are clustered tightly around a straight, flat line.

D The data points are spread evenly over the graph.

---

2 The scatterplot shows the number of minutes it takes for 10 people to run 1 mile, relative to their age in years.



Which best describes the correlation between a person's age and the time it takes him or her to run one mile?

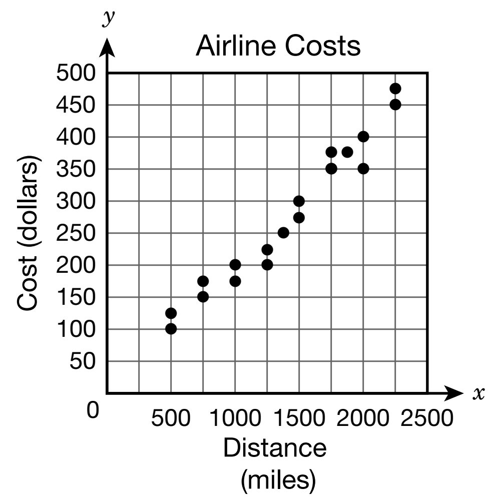
F Linear and positive

G Linear and negative

H Nonlinear and positive

J Nonlinear and negative

3 Below is a scatterplot showing airline costs in dollars relative to the distance traveled in miles.



According to the scatterplot, what is the best estimate for the cost, in dollars, of a flight that covers a distance of 1,700 miles?

A \$150

B \$200

C \$350

D \$450

---

4 A scatterplot shows a positive, nonlinear association. Which relationship does the graph most likely show?

F The area of a square room and the cost to install carpet in the room

G The length of a square room and the cost to install carpet in the room

H The area of a square room and the cost to install a ceiling light in the room

J The length of a square room and the cost to install a ceiling light in the room

Teacher Edition

Measuring Up

Contents

Note: **Eligible standards** are written in **boldface**. R = Readiness standard S = Supporting standard

Chapter 4 Functions

Chapter 5 Geometry and Measurement

Chapter 6 Equations and Inequalities

Chapter 7 Transformations

Chapter 8 Statistics

Chapter 9 Personal Finance

Measuring Up Supplements

Practice Tests
These assessments, written to match the STAAR® blueprints, will help students prepare for the rigor of the STAAR® and are included as blackline masters in the Teacher Edition. They are also available in *Measuring Up Insight®*.

Measuring Up Insight
This Web-based formative assessment program allows teachers to administer pre-made tests (including the STAAR®-emulating Practice Tests), and create and assign custom tests. Analytic reports help monitor student results and customize instruction, review, and remediation.

Measuring Up MyQuest®
Student-centered, standards-based Web-based drill with integrated games makes mastering the TEKS fun. Optional linking to Insight makes practice purposeful.

Lesson Correlation to the Grade 8 Texas Essential Knowledge and Skills

This worktext is customized to the *Texas Essential Knowledge and Skills* and will help you prepare for the *State of Texas Assessments of Academic Readiness (STAAR®)* in Mathematics for Grade 8.

Mathematical process standards are not listed under separate lessons. Because application of mathematical process standards is part of each knowledge statement, these standards are incorporated into instruction and practice throughout the lessons.

Texas Essential Knowledge and Skills	*Measuring Up* Lessons
TEKS 8.2 Number and operations. The student applies mathematical process standards to represent and use real numbers in a variety of forms.	
(A) extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers	1
(B) approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line	2
(C) convert between standard decimal notation and scientific notation	3
(D) order a set of real numbers arising from mathematical and real-world contexts.	2
TEKS 8.3 Proportionality. The student applies mathematical process standards to use proportional relationships to describe dilations.	
(A) generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation	4
(B) compare and contrast the attributes of a shape and its dilation(s) on a coordinate plane	5
(C) use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation.	6
TEKS 8.4 Proportionality. The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope.	
(A) use similar right triangles to develop an understanding that slope, m, given as the rate comparing the change in y-values to the change in x-values, $(y_2 - y_1)/ (x_2 - x_1)$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line	7
(B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship	8
(C) use data from a table or graph to determine the rate of change or slope and y-intercept in mathematical and real-world problems.	9
TEKS 8.5 Proportionality. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions.	
(A) represent linear proportional situations with tables, graphs, and equations in the form of $y = kx$	10
(B) represent linear non-proportional situations with tables, graphs, and equations in the form of $y = mx + b$, where $b \neq 0$	11
(C) contrast bivariate sets of data that suggest a linear relationship with bivariate sets of data that do not suggest a linear relationship from a graphical representation	13
(D) use a trend line that approximates the linear relationship between bivariate sets of data to make predictions	15
(E) solve problems involving direct variation	16
(F) distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$	12
(G) identify functions using sets of ordered pairs, tables, mappings, and graphs	17, 18
(H) identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems	12
(I) write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations.	10, 14
TEKS 8.6 Expressions, equations, and relationships. The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas.	
(A) describe the volume formula $V = Bh$ of a cylinder in terms of its base area and its height	19

Texas Essential Knowledge and Skills	*Measuring Up* Lessons
(B) model the relationship between the volume of a cylinder and a cone having both congruent bases and heights and connect that relationship to the formulas	19
(C) use models and diagrams to explain the Pythagorean Theorem.	20
TEKS 8.7 Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems.	
(A) solve problems involving the volume of cylinders, cones, and spheres	19, 21
(B) use previous knowledge of surface area to make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders	22, 23
(C) use the Pythagorean Theorem and its converse to solve problems	20, 24, 25
(D) determine the distance between two points on a coordinate plane using the Pythagorean Theorem.	25
TEKS 8.8 Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations or inequalities in problem situations.	
(A) write one-variable equations or inequalities with variables on both sides that represent problems using rational number coefficients and constants	26, 27
(B) write a corresponding real-world problem when given a one-variable equation or inequality with variables on both sides of the equal sign using rational number coefficients and constants	28
(C) model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants	26
(D) use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	29, 30
TEKS 8.9 Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to develop foundational concepts of simultaneous linear equations.	
(A) identify and verify the values of x and y that simultaneously satisfy two linear equations in the form $y = mx + b$ from the intersections of the graphed equations.	31
TEKS 8.10 Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts.	
(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane	36
(B) differentiate between transformations that preserve congruence and those that do not	36
(C) explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation	32, 33, 34
(D) model the effect on linear and area measurements of dilated two-dimensional shapes.	35
TEKS 8.11 Measurement and data. The student applies mathematical process standards to use statistical procedures to describe data.	
(A) construct a scatter plot and describe the observed data to address questions of association such as linear, non-linear, and no association between bivariate data	37
(B) determine the mean absolute deviation and use this quantity as a measure of the average distance data are from the mean using a data set of no more than 10 data points	38
(C) simulate generating random samples of the same size from a population with known characteristics to develop the notion of a random sample being representative of the population from which it was selected.	37
TEKS 8.12 Personal financial literacy. The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor.	
(A) solve real-world problems comparing how interest rate and loan length affect the cost of credit	41
(B) calculate the total cost of repaying a loan, including credit cards and easy access loans, under various rates of interest and over different periods using an online calculator	41

Texas Essential Knowledge and Skills		*Measuring Up* Lessons
(C)	explain how small amounts of money invested regularly, including money saved for college and retirement, grow over time	42
(D)	calculate and compare simple interest and compound interest earnings	39, 40
(E)	identify and explain the advantages and disadvantages of different payment methods	43
(F)	analyze situations to determine if they represent financially responsible decisions and identify the benefits of financial responsibility and the costs of financial irresponsibility	43
(G)	estimate the cost of a two-year and four-year college education, including family contribution, and devise a periodic savings plan for accumulating the money needed to contribute to the total cost of attendance for at least the first year of college.	44

Lesson 6

Algebraic Representations of Dilations

R 8.3(C) Use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation.

Understand the TEKS

A **dilation** of a figure is an **enlargement** or a **reduction** that creates an **image** similar to the original figure. The ratio of an original side length to its corresponding image side length is constant. This ratio is the **scale factor**.

Dilations on a coordinate grid are performed by multiplying each *x*-coordinate and *y*-coordinate by the scale factor. This can be represented algebraically by $(x, y) \rightarrow (sx, sy)$, where s is the scale factor.

Words to Know
- dilation
- enlargement
- reduction
- image
- scale factor

Guided Instruction

Problem 1

Transformation $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$ is applied to quadrilateral *FGHJ*, which has coordinates $F(-3, -1)$, $G(-6, 2)$, $H(1, 1)$, and $J(4, -3)$. What are the coordinates of $F'G'H'J'$?

Multiply the coordinates of the vertices by the scale factor.

Step 1 Determine the transformation.

A dilation is represented as $(x, y) \rightarrow (sx, sy)$, where s is the scale factor. Because this transformation is shown as $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$, it is a dilation with a scale factor $\frac{1}{2}$.

Step 2 Write the coordinates of the vertices of *FGHJ*.

F $(-3, -1)$, G $(-6, 2)$, H $(1, 1)$, J $(4, -3)$

Step 3 Multiply the coordinates of the vertices of *FGHJ* by the scale factor to find the coordinates of the vertices of $F'G'H'J'$.

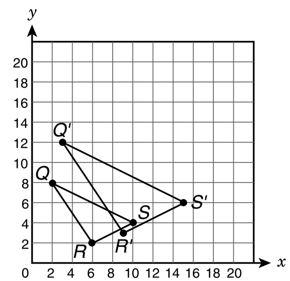
$F(-3, -1)$	$F\left[\left(-3 \cdot \frac{1}{2}\right), \left(-1 \cdot \frac{1}{2}\right)\right]$	$F'\left(-1\frac{1}{2}, -\frac{1}{2}\right)$
$G(-6, 2)$	$G\left[\left(-6 \cdot \frac{1}{2}\right), \left(2 \cdot \frac{1}{2}\right)\right]$	$G'(-3, 1)$
$H(1, 1)$	$H\left[\left(1 \cdot \frac{1}{2}\right), \left(1 \cdot \frac{1}{2}\right)\right]$	$H'\left(\frac{1}{2}, \frac{1}{2}\right)$
$J(4, -3)$	$J\left[\left(4 \cdot \frac{1}{2}\right), \left(-3 \cdot \frac{1}{2}\right)\right]$	$H'\left(2, -1\frac{1}{2}\right)$

Solution

What are the coordinates of $F'G'H'J'$? The resulting coordinates are reduced by a factor of $\frac{1}{2}$ to $F'\left(-1\frac{1}{2}, -\frac{1}{2}\right)$, $G'(-3, 1)$, $H'\left(\frac{1}{2}, \frac{1}{2}\right)$, $J'\left(2, -1\frac{1}{2}\right)$.

Problem 2

What is the algebraic rule that defines the dilation from $\triangle QRS$ to $\triangle Q'R'S'$? Present the answer in decimal form.



Determine the scale factor.

Step 1 Write the coordinates of the original figure.

Q (2, 8), R (6, 2), S (10, 4)

Step 2 Write the coordinates of the image.

Q' (3, 12), R' (9, 3), S' (15, 6)

Step 3 Determine the scale factor with a coordinate pair.

$Q(2, 8)$ $Q[(2 \cdot$ 1.5 $), (8 \cdot$ 1.5 $)]$ $Q'(3, 12)$

Step 4 Write the algebraic rule for the dilation.

$(x, y) \rightarrow ($ 1.5 x, 1.5 $y)$

Solution

What is the algebraic rule that defines the dilation from $\triangle QRS$ to $\triangle Q'R'S'$?

$(x, y) \rightarrow (1.5x, 1.5y)$

Critical Thinking

Solve each problem.

Geography Connection

1. A geography class studies the locations of various countries in Africa using a poster-sized map. The students then take a test using a map shown on an 8.5- by 11-inch sheet of paper. Explain how the dilation $(x, y) \rightarrow (5x, 5y)$ could transform one map to the other if the same corner of each map were aligned as an origin on a coordinate grid. Which would be the original figure, and which would be the image?

Sample answer: Because the scale factor in the dilation rule is 5, which is greater than 1, it is an enlargement. Therefore, the dilation is transforming the original figure, which is the map on the paper, to the poster-sized map image.

Real World Connection

2. A local community is building a skate park on a rectangular plot of land. The skate area will be surrounded by seating. A contractor suggests that they eliminate some surrounding seating and change the original plans of the skate area by $(x, y) \rightarrow (\frac{1}{4}x, \frac{1}{4}y)$ so more people can skate at a time. Explain whether the contractor's suggestion is reasonable.

The contractor's answer is not reasonable because the transformation shown is a dilation with a scale factor of one-fourth, which is less than 1. This means the dilation is a reduction. The skate area will be smaller.

Collaborative Learning

3. Work with a partner to draw triangles on a coordinate grid with vertices at $(n, n + 1)$, $(n, n - 1)$, and $(n + 1, n - 1)$ for three different values of n. How does the value of n affect the original figures? How does it affect the final image when a dilation of $(x, y) \rightarrow (2x, 2y)$ is applied?

Answers will vary but should include the following: The value of n affects only the location of the original figure; however, it affects the location and size of the final image.

Elevate

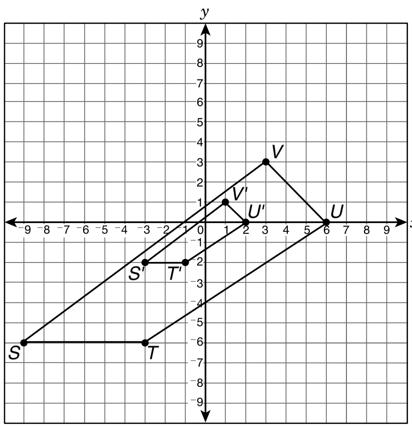
4. Could you dilate a square on a coordinate grid as $(x, y) \rightarrow (0.5x, 0.5y)$ by instead applying $(x, y) \rightarrow (x - a, y - a)$ where $a = 0.5x$? Explain why or why not.

Sample answer: No. If $a = 0.5x$, then $x - a$ would give $0.5x$. However, $y - 0.5x$ would only give $0.5y$ if $x = y$, and that will not always be true in a square on a coordinate grid.

★ Practice

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 What values of m, n, p, *and* q would result in the dilation shown, where $(x, y) \rightarrow \left(\left(\frac{m}{n}\right)x, \left(\frac{p}{q}\right)y\right)$?



A $m = 2$, $n = 2$, $p = 6$, $q = 6$

B $m = 3$, $n = 9$, $p = 1$, $q = 3$

C $m = 12$, $n = 4$, $p = 12$, $q = 4$

D $m = 4$, $n = 16$, $p = 4$, $q = 16$

2 Two of the vertices of $\triangle SLM$ are at (5, 10) and (250, 500). The corresponding vertices of $\triangle KNR$ are at (0.2, 0.4) and (10, 20), but it is unknown which sets of coordinate pairs correspond with each other. Which of the following could be a rule of the dilation relating the triangles?

F $(x, y) \rightarrow (1{,}250x, 1{,}250y)$

G $(x, y) \rightarrow (0.5x, 0.5y)$

H $(x, y) \rightarrow (25x, 25y)$

J $(x, y) \rightarrow (2x, 2y)$

3 A dilation of $(x, y) \rightarrow (0.2x, 0.2y)$ is applied to a triangle. The image coordinates are $(4t, 3t)$, $(-7t, t)$, and $(10t, -2t)$. Which of the following is a coordinate of the original figure?

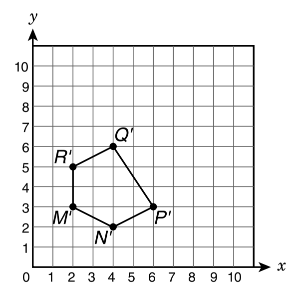
A $(50t, -10t)$

B $(0.8t, 0.6t)$

C $(-7.2t, 1.2t)$

D $(3.8t, 2.8t)$

4 Point Q in figure $MNPQR$ is at $\left(\frac{1}{2}, \frac{3}{4}\right)$. The coordinate grid shows the image after dilation.



Which rule represents the dilation?

F $(x, y) \rightarrow (8x, 8y)$

G $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$

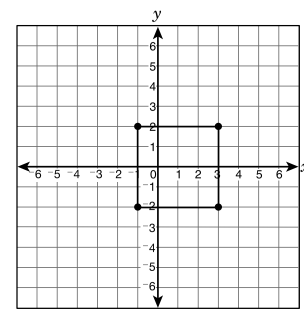
H $(x, y) \rightarrow \left(\frac{1}{8}x, \frac{1}{8}y\right)$

J $(x, y) \rightarrow \left(\frac{2}{3}x, \frac{2}{3}y\right)$

★ Assessment

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 The transformation rule $(x, y) \rightarrow (4x, 4y)$ is applied to the figure shown.



Which are the final coordinates?

A (0.75, 0.5), (0.75, −0.5), (−0.25, −0.5), (−0.25, 0.5)

B (12, 8), (12, −8), (−12, −8), (−12, 8)

C (12, 8), (12, −8), (−4, −8), (−4, 8)

D (0.75, 0.5), (0.75, −0.5), (−0.75, −0.5), (−0.75, 0.5)

2 A figure has vertices (x, y), $(x + 2, y)$ and $(x + 6, y + 2)$. If it is dilated by $(x, y) \rightarrow (3x, 3y)$, what are the new coordinates?

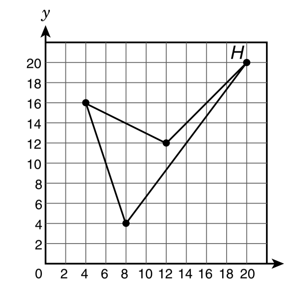
F $(3x, 3y)$, $(3x + 6, 3y)$, $(3x + 18, 3y + 6)$

G $(x + 3, y + 3)$, $(x + 5, y + 3)$, $(x + 9, y + 5)$

H $(3x, 3y)$, $(3x + 2, 3y)$, $(3x + 6, 3y + 2)$

J $(3x, 3y)$, $(3x, 3y)$, $(3x, 3y)$, $(3x, 3y)$

3 If $(x, y) \rightarrow (4ax, 4ay)$, for what value of a would the rightmost point of the resulting image have an x-value of 10?



A $\frac{4}{10}$

B $\frac{1}{8}$

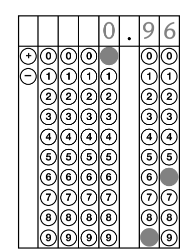
C $\frac{1}{2}$

D $\frac{10}{4}$

4 Three dilations are applied to a figure in the order shown.
$(x, y) \rightarrow (0.8x, 0.8y)$
$(x, y) \rightarrow (0.06x, 0.06y)$
$(x, y) \rightarrow (20x, 20y)$

What value of r provides a single rule that would give the same image as the three rules shown? Write your answer in decimal form. $(x, y) \rightarrow (rx, ry)$

Record your answer and fill in the bubbles on the following grid. Be sure to use the correct place value.



Lesson 36 Properties of Transformations

8.10(A) Generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

8.10(B) Differentiate between transformations that preserve congruence and those that do not.

Understand the TEKS

Words to Know
transformations
congruence
orientation

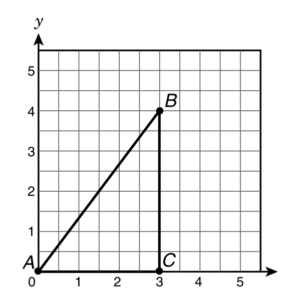
The **transformations** that describe movement of a figure are translations, reflections, rotations, and dilations.

Translations, reflections, and rotations are **congruence** transformations because the pre-image and image are congruent. While a reflection results in congruence, it is the only transformation that does not preserve **orientation**, or the order of the vertices.

Dilations with a scale factor other than 1 result in an enlargement or a reduction of the figure, so this is not a congruence transformation. The difference in the effect on congruence can be recognized in the algebraic form of the transformations. Dilations are the only transformations that incorporate a scale factor other than 1 or -1 in the x-coordinate and y-coordinate of the image.

Guided Instruction

Problem 1 Triangle ABC undergoes a transformation on the coordinate grid. What are the potential effects on congruence and orientation?



Step 1 Analyze the effects of a translation.

A translation would move the triangle according to $(x, y) \rightarrow (x + a, y + b)$.

The resulting image ___will___ be congruent and the orientation will be ___the same___.

Step 2 Analyze the effects of a reflection.

A reflection would move the triangle according to $(x, y) \rightarrow (-x, y)$ or $(x, y) \rightarrow (x, -y)$.

The resulting image ___will___ be congruent and the orientation will be ___different___.

Step 3 Analyze the effects of a rotation.

A rotation would move the triangle according to $(x, y) \rightarrow (y, -x)$, $(x, y) \rightarrow (-x, -y)$, $(x, y) \rightarrow (-y, x)$, or $(x, y) \rightarrow (x, y)$.

The resulting image ___will___ be congruent and the orientation will be ___the same___.

Step 4 Analyze the effects of a dilation.

A dilation would move the triangle according to $(x, y) \rightarrow (sx, sy)$.

The resulting image __will not__ be congruent, if the dilation factor has a value other than 1, and the orientation will be __the same__.

Solution

What are the potential effects on congruence and orientation?

If the transformation is a translation, reflection, or rotation, the image will be congruent. If it is a dilation, the image will not be congruent. The orientation will stay the same unless the transformation is a reflection.

Problem 2

A transformation is performed on a figure such that $(x, y) \rightarrow (ax, by)$. What must be true for the transformation to preserve congruence?

Step 1 Each transformation that preserves congruence has an algebraic rule with coefficients of ±__1__ for x and y. So, $|a|$ must have a value of __1__.

Step 2 Notice that you do not need to state a relationship between a and b, because a dilation is the only transformation that does not preserve congruence, and if b was a positive number other than 1, a would be also.

Solution

What must be true for the transformation to preserve congruence?

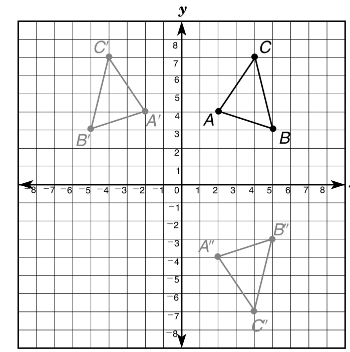
The absolute value of a must equal 1.

Critical Thinking

Solve each problem.

Elevate

1. Consider $\triangle ABC$ with vertices at $A(2, 4)$, $B(5, 3)$, $C(4, 7)$. First, graph $\triangle A'B'C'$, the reflection of $\triangle ABC$ across the y-axis. Then, graph $\triangle A''B''C''$, the reflection of $\triangle ABC$ across the x-axis. Describe how the image triangles are like the original triangle, and how they are different.



Sample answer: The image triangles and the pre-image triangle are congruent. The orientations of the image triangles are the same as each other (clockwise order for the vertices), but they are opposite from the pre-image triangle. All the triangles are in different quadrants.

Real World Connection

2. An architect is designing the layout of a house on a coordinate grid that represents a plot of land. She has the floor plan drawn with the living room on the northeast side of the house. She decides which part of the house is facing north without changing the orientation of the house. What transformation can she do? Explain your reasoning.

Sample answer: The transformations that will change which part is at the top are reflection and rotation. Reflection changes orientation, so she can only use a rotation.

Discuss

3. Work with a partner to analyze whether a translation followed by a dilation with a scale factor not equal to 1 could have the same final image as a rotation. Use algebraic representations of the transformations to explain your reasoning. Record examples and findings in your math journal. Share your results with the class.

Answers will vary, but should include the statement that they could not have the same final image because a dilation is represented as $(x, y) \rightarrow (sx, sy)$ where $s \neq 1$, so the image will not be congruent.

★ Practice

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 A figure is plotted in Quadrant I on a coordinate grid. Which of the following could result in a figure that is in the same location and looks identical to the original figure? Note: $a \neq 0$ and $b \neq 0$.

A $(x, y) \rightarrow (x + a, y + b)$

B $(x, y) \rightarrow (-x, y)$

C $(x, y) \rightarrow (ax, ay)$

D $(x, y) \rightarrow (-x, -y)$

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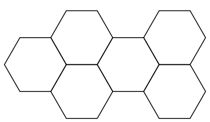
2 A regular tessellation is a collection of one or more congruent regular polygons with no gaps or overlaps.



Which transformation could not be used to create the tessellation?

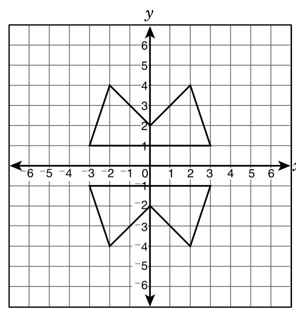
F Dilation

G Reflection

H Translation

J Rotation

3 The coordinate grid shows the pre-image and image of a transformation.



Which of the following statements must be true about the figures?

A The figures may or may not be congruent.

B The orientation must be different.

C The orientation must be the same.

D The figures must be congruent.

---

4 What effect does a reflection have on the perimeter and area of a triangle?

F The perimeter must change.

G The area must stay the same.

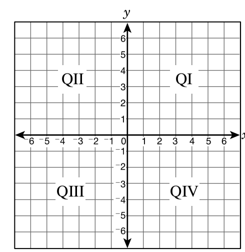
H The perimeter may increase.

J The area may decrease.

★ Assessment

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 Which algebraic representation could be a transformation that takes the pre-image from Quadrant IV to an image that is congruent, has the same orientation, and is in the same quadrant? Note: $a > 1$ and $b \neq 0$.



A $(x, y) \rightarrow (-y, x)$

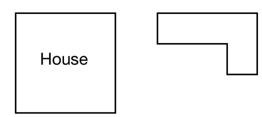
B $(x, y) \rightarrow (-x, y)$

C $(x, y) \rightarrow (ax, ay)$

D $(x, y) \rightarrow (x + a, y + b)$

---

2 Jerome is buying fencing to build a dog run in his backyard as shown.



He decides to move the dog run according to the transformation $(x, y) \rightarrow (y, -x)$. How is the cost of his project material affected?

F The cost increases.

G The cost decreases.

H The cost stays the same.

J The values for x and y must be known to determine the effect.

3 A figure has vertices labeled alphabetically from M, in the counterclockwise direction. Which single transformations result in an image with vertices labeled alphabetically from M in the clockwise direction?

I. Rotation 180° clockwise
II. Rotation 270° clockwise
III. Reflection across the x-axis
IV. Reflection across the y-axis

A I only

B III and IV only

C I and II only

D I, II, III, and IV

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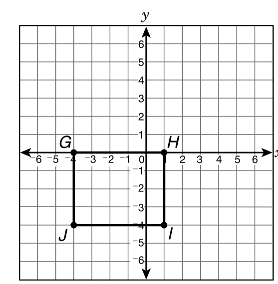
4 Rhonda claims that she can reflect the rectangle across the y-axis to get an identical rectangle to the one created by the transformation $(x, y) \rightarrow (x + 3, y + 0)$.



Is she correct? Why or why not?

F Yes, the rectangles are congruent with identical orientation.

G No, one rectangle changes orientation.

H No, the rectangles are not congruent.

J No, the rectangles are not congruent and have different orientations.

Lesson 37 Scatterplots

Ⓢ 8.11(A) Construct a scatterplot and describe the observed data to address questions of association such as linear, non-linear, and no association between bivariate data.

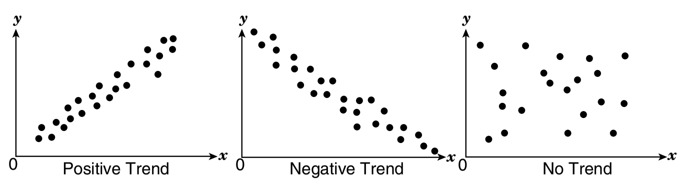
8.11(C) Simulate generating random samples of the same size from a population with known characteristics to develop the notion of a random sample being representative of the population from which it was selected.

Understand the TEKS

Scatterplots help us determine whether there is a relationship between two quantities. A scatterplot is a graph of individual data points relating two different types of values (such as width and height) for a set of data. A pattern of data points on a scatterplot is called a **trend**, an **association**, or a **correlation**. Sometimes the data points of a scatterplot may be approximated by a straight line, in which case the quantities have a **linear association**. If the slope of this line is positive, the association between the two variables is positive. If the slope is negative, the association is negative. If the data points in a scatterplot cluster closely to the line, the association is considered strong. If the data points vary in distance from the general line, the association is considered weak.

If the data can be approximated by a line that is not straight, the association is nonlinear. Some data sets can show no trend between the two variables.

Words to Know
scatterplot
trend
association
correlation
linear association
random sample
population



Statistical data is often represented using scatterplots. When very large populations are studied, such as all teenagers in the United States, **random samples** are taken. Random sampling enables a **population** to be studied that would otherwise be unreasonable to study due to the large sample size. It is not reasonable to survey all teenagers in the United States about their favorite musical artist. Random sampling within the population is much more feasible as a members of the population are chosen randomly to provide data for a study. Random sampling provides accurate results for an entire population given that the sample taken is truly random and sufficiently large in number.

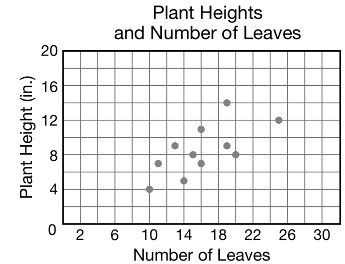
Guided Instruction

Problem

Mr. Connor's science class is studying plant growth. They want to want to learn more about the relationship between the height of a plant and the number of leaves on the plant. They measured the height of 11 plants, counted the number of leaves on each plant, and recorded the data in the table below. Plot the data in the table. What is the relationship between the height of the plant and the number of leaves?

Step 1 Plot the data points.

Number of Leaves	Height of Plant (in.)
10	4
11	7
13	9
14	5
15	8
16	7
16	11
19	9
19	14
20	8
25	12



Step 2 Interpret the scatterplot. The data points show that plant height tends to increase as the number of leaves increases.

Step 3 Determine if the association is linear or nonlinear.

Do the points form a reasonably straight line? yes

Step 4 Determine if the association is weak or strong.

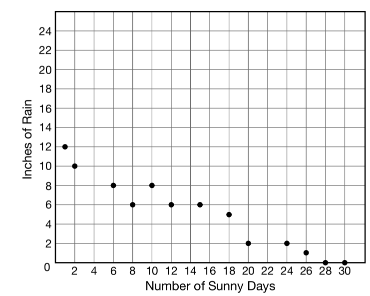
The dots are clustered closely to form a line, so the association is strong.

Solution

What is the relationship between the height of the plant and the number of leaves?
The data show a strong, positive, and linear trend.

Another Example

The scatterplot shows the number of sunny days and the number of inches of rainfall in a city during one month in different years.



Determine whether there is an association between the variables. If so, describe the association.

About how many inches of rain would you expect to fall in a year that has 4 sunny days in the month?

As the number of days of sunshine increases, the number of inches of rainfall tends to decrease.

The dots form a straight line, so the association is linear.

Look at the number of inches of rainfall for fewer than 4 sunny days and more than 4 sunny days.

When there were 2 sunny days, there were 10 inches of rainfall.

When there were 6 sunny days, there were 8 inches of rainfall.

If there were 4 days of sunshine, there would be between 10 inches and 8 inches of rain.

Critical Thinking

Solve each problem.

Collaborative Learning

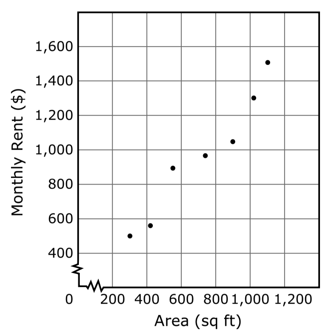
1. Work in groups of 2 or 3 as directed by your teacher. Survey your classmates. Ask each classmate for their height and shoe size. Create a scatterplot to show the data. Determine if an association exists between a student's height and shoe size. If so, describe the association. Can you use your scatterplot to make any conclusions about a population? Explain.

If the data do not show an association, suggest that students create separate data plots for boys and girls to account for differences in sizing. Students should recognize that although their class is not a random sample, the data can be used to represent the population of local eighth-grade students.

Elevate **2.** The scatterplot at the right shows the relationship between the sizes (in square feet) of apartments in a certain neighborhood of a city and the price of those apartments.

Describe any association shown in the scatterplot.

Using the scatterplot, predict the cost of an apartment that has an area of 500 square feet.
Show or explain your work.



I saw a positive, linear association in the data. I read the data point for about 400 square feet, which cost about $550, and the data point for about 550 square feet, which cost about $900. So, an apartment with an area of 500 square feet would cost between $550 and $900, or about $750.

Media Connection **3.** Search the Internet to find an example of a published scatterplot. On a poster, describe what data the scatterplot represents, what trends it shows, and three predictions that you can make from the scatterplot. Use colored markings to show how you found your predictions based on the graph. Write one quiz question about your scatterplot, with the answer on the other side, so that other students may quiz themselves using your graph.

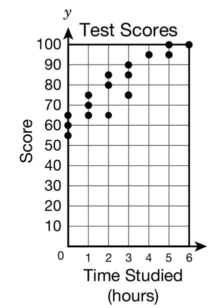
Elevate **4.** The association shown in a scatterplot can be described as strong or weak. What would a strong association look like? What would a weak association look like? Explain.

A strong association would have data points that are closely clustered around the shape of a line; a weak association would have points farther away from the line.

★ Practice

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 The scatterplot below shows the test scores and number of hours spent studying for 16 students.



Which statement best describes the correlation between the study times and the test scores?

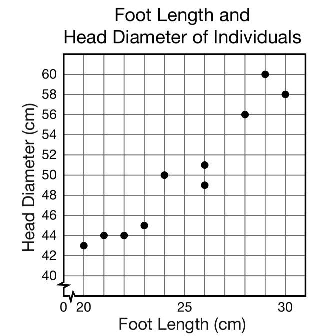
A There is a strong positive correlation between the time spent studying and the test score.

B There is a negative correlation between the time spent studying and the test score.

C There is a weak positive correlation between the time spent studying and the test score.

D There is a weak negative correlation between the time spent studying and the test score.

2 The scatterplot below shows the relationship between foot length and head diameter of individuals.



If a person has a head diameter of approximately 54 centimeters, what would be the most reasonable foot length for that individual based on the data provided?

F 21 cm

G 23 cm

H 27 cm

J 30 cm

3 If you graphed data that compared the price of a movie ticket to annual sales for that movie, what would you expect the scatterplot to show?

A A strong positive correlation

B A weak negative correlation

C A weak positive correlation

D No correlation

★ Assessment

DIRECTIONS Read each question. Then circle the letter for the correct answer.

1 A scatterplot shows the relationship between the value of a rare comic book, in dollars, and the number of years since it was purchased. What would you expect the scatterplot to look like?

A The data points are clustered tightly around a straight line with a positive slope.

B The data points are clustered tightly around a straight line with a negative slope.

C The data points are clustered tightly around a straight, flat line.

D The data points are spread evenly over the graph.

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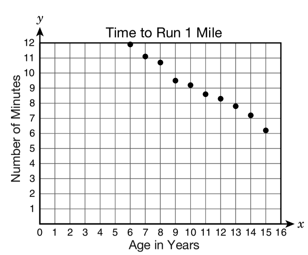
2 The scatterplot shows the number of minutes it takes for 10 people to run 1 mile, relative to their age in years.



Which best describes the correlation between a person's age and the time it takes him or her to run one mile?

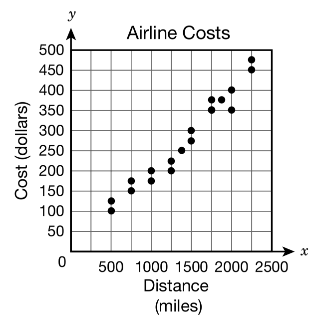
F Linear and positive

G Linear and negative

H Nonlinear and positive

J Nonlinear and negative

3 Below is a scatterplot showing airline costs in dollars relative to the distance traveled in miles.



According to the scatterplot, what is the best estimate for the cost, in dollars, of a flight that covers a distance of 1,700 miles?

A \$150 **C** \$350

B \$200 **D** \$450

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4 A scatterplot shows a positive, nonlinear association. Which relationship does the graph most likely show?

F The area of a square room and the cost to install carpet in the room

G The length of a square room and the cost to install carpet in the room

H The area of a square room and the cost to install a ceiling light in the room

J The length of a square room and the cost to install a ceiling light in the room